1. Details of Module and its structure

Module Detail	
Subject Name	Physics
Course Name	Physics 01 (Physics Part-1, Class XI)
Module Name/Title	Unit 5, Module 5, Torque Chapter 7, System of particles and rotational motion
Module Id	Keph_10705_eContent
Pre-requisites	Kinematics, laws of motion, basic vector algebra
Objectives	After going through this lesson, the learners will be able to : • **Understand** the meaning of 'torque' • **Relate** torque as Rotational analogue of force and establish its SI unit • **Derive** an expression relating torque and angular acceleration for a system of particles.
Keywords	Torque, Moment of force, Couple, right hand palm rule, Torque due to a couple

2. Development Team

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1. UNIT SYLLABUS

Unit V: Motion of System of Particles and Rigid body

Chapter 7: System of particles and Rotational Motion

Centre of mass of a two-particle system; momentum conservation and centre of mass motion. Centre of mass of a rigid body; Centre of mass of a uniform rod.

Moment of a force; torque; angular momentum; law of conservation of angular momentum and its applications.

Equilibrium of rigid bodies; rigid body rotation and equations of rotational motion; comparison of linear and rotational motions.

Moment of inertia; radius of gyration; values of moments of inertia for simple geometrical objects (no derivation). Statement of parallel and perpendicular axes theorems and their applications.

2. MODULE-WISE DISTRIBUTION OF UNIT SYLLABUS 8 Modules

The above unit is divided into eight modules for better understanding.

Module 1	• Rigid body • Centre of mass • Distribution of mass • Types of motion: Translatory, circulatory and rotatory
Module 2	• Centre of mass • Application of centre of mass to describe motion • Motion of centre of mass
Module 3	• Analogy of circular motion of a point particle about a point and different points on a rigid body about an axis • Relation v = rω • Kinematics of rotational motion
Module 4	• Moment of inertia • Difference between mass and moment of inertia • Derivation of value of moment of inertia for a lamina about a vertical axis perpendicular to the plane of the lamina • unit • Radius of gyration • Perpendicular and parallel axis theorems
Module 5	• Torque • Types of torque • Dynamics of rotator motion • $m = I\alpha$
Module 6	• Equilibrium of rigid bodies • Condition of net force and net torque • Applications
Module 7	• Law of conservation of angular momentum and its applications.

Module 8	• Rolling on plane surface • Rolling on Horizontal • Rolling on inclined surface • Applications

Module 5

3. WORDS YOU MUST KNOW

Let us remember the words we have been using in our study of this physics course.

- Rigid body: An object for which individual particles continue to be at the same separation over a period of time
- Point object: A point object is much smaller than the distance due to change in its position.
- Distance travelled: The length of the path covered by a body from initial position to a final position. Its SI unit is m and it can be zero or positive.
- Displacement: The distance an object has moved from its starting position moves in a particular direction. Its SI unit is m and it can be zero, positive or negative.
- Speed: Rate of change of position and its unit m/s.
- Average speed: Average speed $=\frac{\text{Total path length}}{\text{Total time interval}}$

  Its unit is m/s.
- Velocity (v): Rate of change of position in a particular direction and its unit is m/s.
- Instantaneous velocity The velocity at an instant is defined as the limit of the average velocity as the time interval Δt becomes infinitesimally small velocity at any instant of time
- Uniform motion: object covers equal distance in equal interval of time
- Non uniform motion: object covers unequal distance in equal interval of time
- Acceleration (a): rate of change of velocity with time and its unit is m/s^2. Velocity may change due to change in its magnitude or change in its direction or change in both magnitude and direction.
- Constant acceleration: Acceleration which remains constant.

- Momentum (p): The impact capacity of a moving body is mv and its unit is kg m/s.
- Force (F): Something that changes the state of rest or uniform motion of a body. Unit of force is Newton. It is a vector, as it has magnitude which tells us the strength or magnitude of the force and the direction of force is very important
- Constant force: A force for which both magnitude and direction remain the same with passage of time
- Variable force: A force for which either magnitude or direction or both change with passage of time
- External unbalanced force: A single force or a resultant of many forces that act externally on an object.
- Dimensional formula: An expression which shows how and in which way the fundamental quantities like, mass (M), length (L) and time (T).
- Kinematics: Study of motion without involving the cause of motion
- Dynamics: Study of motion along with the cause producing the motion
- Vector: A physical quantity that has both magnitude and direction .displacement is a vector, force is a vector, acceleration is a vector etc.
- Vector algebra: Mathematical rules of adding, subtracting and multiplying vectors
- Resolution of vectors: A vector can be resolved in two mutually perpendicular directions. We used this for vector addition and in our study of motion in 2 and 3 dimensions.
- Dot product: two vectors on multiplication yield a scalar quantity. Dot product of vector A and B: A.B= $|A||B|cos\theta$ where θ is the angle between the two vectors. Dot product is a scalar quantity and has no direction. It can also be taken as the product of magnitude of A and the component of B along A or product of B and component of A along B.
- Work: Work is said to be done by an external force acting on a body if it produces displacement W= F.S cosθ, where work is the dot product of vector F(force) and Vector S (displacement) and θ is the angle between them . Its unit is joule and dimensional formula is ML^2T^{-2}. It can also be stated as the product of component of the force in the direction of displacement and the magnitude of

displacement. Work can be done by constant or variable force and work can be zero, positive or negative.

- Energy: The ability of a body to do work. Heat, light, chemical, nuclear, mechanical are different types of energy. Energy can never be created or destroyed it only changes from one form to the other.
- Kinetic Energy: The energy possessed by a body due to its motion = ½ mv^2, where 'm' is the mass of the body and 'v' is the velocity of the body at the instant its kinetic energy is being calculated.
- Conservative force: A force is said to be conservative if the work done by the force in displacing a body from one point to another is independent of the path followed by the particle and depends on the end points. Example: gravitational force.
- Non- conservative forces: **A** force is said to be non-conservative if: the work done by it on an object depends on the path and the work done by it through any round trip is not zero. Example: friction.
- Work Energy theorem: Relates work done on a body to the change in mechanical energy of a body i.e., $W = F.S = \frac{1}{2} mV_f^2 - \frac{1}{2} mV_i^2$
- Conservation of mechanical energy: Mechanical energy is conserved if work done is by conservative forces.
- Potential energy due to position: Work done in raising the object of mass m to a particular height (distance less than radius of the earth) = 'mgh'.
- Collision: Sudden interaction between two or more objects. We are only considering two body collisions.
- Collision in one dimension: Interacting bodies move along the same straight path before and after collision.
- Elastic collision: Collision in which both momentum and kinetic energy is conserved.
- Inelastic collision: Momentum is conserved but kinetic energy is not conserved.
- Coefficient of restitution: The ratio of relative velocity after the collision and relative velocity before collision. Its value ranges from 0-1.

4 INTRODUCTION

In the earlier modules we primarily considered the motion of a single particle. A particle is represented as a point mass. It has practically no size. We applied the results of our study even to the motion of bodies of finite size, assuming that motion of such bodies can be described in terms of the motion of a particle. Any real body which we encounter in daily life has a finite size. In dealing with the motion of extended bodies (bodies of finite size) often the idealized model of a particle is inadequate. In this chapter we shall try to go beyond this inadequacy. We shall attempt to build an understanding of the motion of extended bodies. An extended body, in the first place, is a system of particles. We shall begin with the consideration of motion of the system as a whole. The centre of mass of a system of particles will be a key concept here. We shall discuss the motion of the centre of mass of a system of particles and usefulness of this concept in understanding the motion of extended bodies.

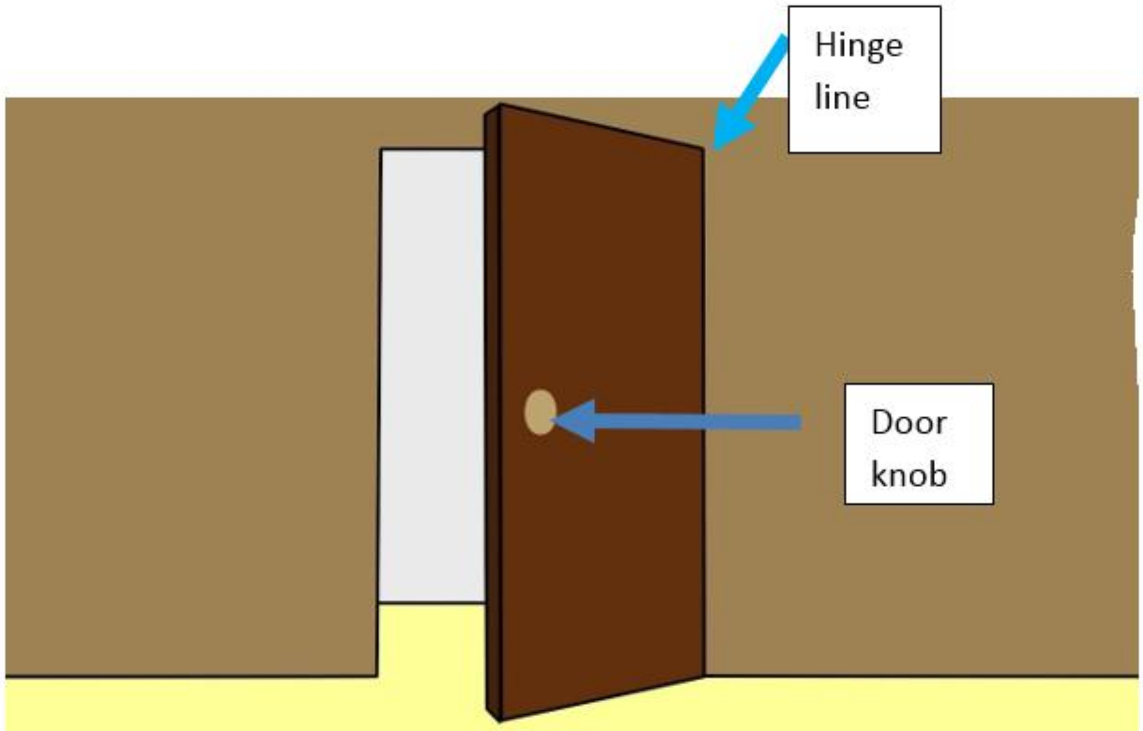
A large class of problems with extended bodies can be solved by considering them to be rigid bodies. Ideally a rigid body is a body with a perfectly definite and unchanging shape. The distances between all pairs of particles of such a body do not change. It is evident from this definition of a rigid body that no real body is truly rigid, since real bodies deform under the influence of forces. But in many situations the deformations are negligible.

In a number of situations involving bodies such as wheels, tops, steel beams, molecules and planets on the other hand, we can ignore that they warp, bend or vibrate and treat them as rigid.

5 TORQUE

We know that force is needed to change the state of rest or uniform translational motion of a body, i.e. to produce linear acceleration. **We may then ask, what is the analogue of force in the case of rotational motion?** Only does it include force or some more factors are needed.

Let us take the example of opening or closing of a door. A door is a rigid body which can rotate about a fixed vertical axis passing through the hinges. What makes the door rotate? It is clear that unless a force is applied, the door does not rotate. Hence, **force is essential for making an object at rest to start rotating**



Notice the door the hinge line, door knob

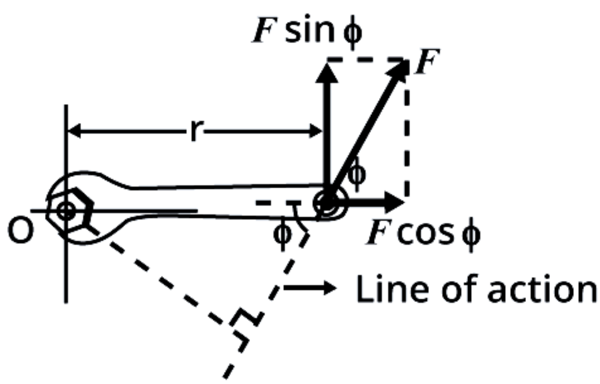
Why do they place door knobs so close to the edge?

But do all forces do this job equally efficiently for us?

A force, applied along the hinge line, cannot produce any rotation at all. Similarly, by applying forces at different points and at different angles, one can find out that when a force of a given magnitude is applied at right angles to the hinged door at its outer edge it becomes most effective in producing rotation. It follows that it is not the force alone, but how and where the force is applied, that is also very important in rotational motion. We can, in very simple words, say the rotational analogue of force is the moment of force or torque. The word “torque” has been derived from a latin word “torquere”, which means to twist. The torque of a force, depends not only on the force but also on its point and direction of application.

The tendency of a force to produce a rotational effect in an object about some axis is measured by a vector quantity called torque τ.

Consider the wrench pivoted on the axis through O in the figure.



The applied force F acts at an angle Ø to the horizontal. We define the magnitude of the torque associated with the force F, by the expression:

$$T = r\,F\,\sin\emptyset = F\,d$$

where r is the distance between the pivot point and the point of application of F and d is the perpendicular distance from the pivot point to the line of action of F.

The line of action of a force is an imaginary line extending out on both ends of the vector, representing the force.

This quantity d is the perpendicular distance from the pivot point to the line of action of F is also called the moment arm (or lever arm) of Force F.

In the above Figure one can see, the only component of F that tends to cause rotation is

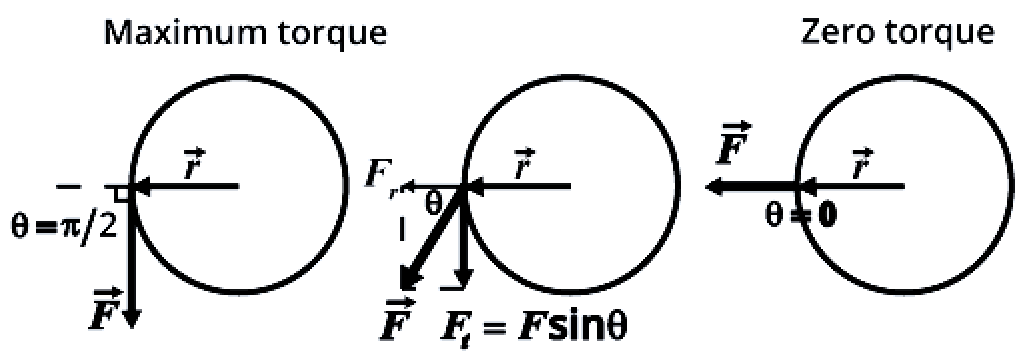
$F_T = F\sin\emptyset$, the component perpendicular to r.

It is also called the tangential component. The other component $F_R = F\cos\emptyset$, is called the radial component, because it passes through O. This component has no tendency to produce rotation.

We say so because as we have noted above, a force, applied along the hinge line, cannot rotate the door at all.

It is very important that you recognize that torque is defined only through the specification of the reference axis, about which rotation is to take place. Torque is the product of a force and the moment arm of that force, the moment arm is defined only in terms of the axis of rotation.

6 MOMENT OF FORCE



The upper set of diagrams shows the dependence of torque on the angle θ. Maximum torque occurs when the force F is at right angles to r (line joining the pivot point and point of application of force) i.e. when $\theta = 90°$. The central figure shows that it is only the tangential component of F equal to F sin θ which is responsible for torque. While in the torque is zero or minimum when the force F is along the line joining the pivot point and point of application of force i.e. when $\theta = 0°$.

In short one has to keep in mind the following points when moment of a force is to be determined. They are

1. **Where is the pivot point?**
2. **What was the magnitude of the force applied?**
3. **How far from the pivot point was the force applied?**

4. **What was the angle between the vector (the position vector of the point of application of the force with respect to the pivot point) and the direction of applied force?**

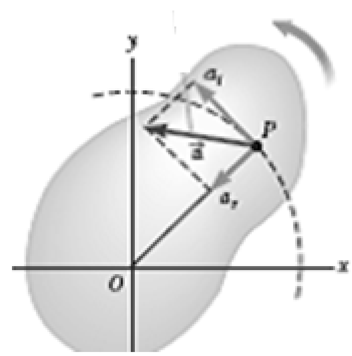
UNIT OF TORQUE:

The symbol **τ** stands for the Greek letter **tau**. The SI unit of torque is Nm or Newton-metre.

It is interesting to note that torque and work share the same unit and dimensional formula but they are very different in nature, and meaning, from each other!

7 RELATION BETWEEN TORQUE, ANGULAR ACCELERATION AND MOMENT OF INERTIA

Consider an object rotating with an angular acceleration α about the z axis, as shown in the figure given below. Consider a particle P, of mass m rotating in a circle of radius r under the influence of a tangential force F_T and a radial force F_R as shown in Figure. As we know the radial force is necessary to keep the particle moving in its circular path, while the tangential force provides a tangential acceleration a_T .The torque about the center of the circle due to F_T is because the tangential acceleration is related to the angular acceleration through the relationship



$$a_T = \alpha\, r$$

and the contribution of radial force is zero as the force passes through the axis of rotation.

The torque can be expressed as

$$\tau = r\, F_T = r\, m\, a_T = m\, r^2\, \alpha$$

The above expression can be extended to the whole body, we can then write

$$\tau = \left(\sum_{i=1}^{N} m_i r_i^2\right)\alpha$$

$$\tau = I\,\alpha$$

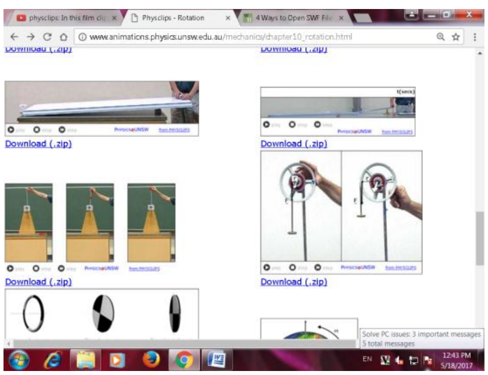
Here, $I = \sum_{i=1}^{N} m_i r_i^2$ is the moment of inertia of the rotating object about the z axis passing through the origin. Hence, the torque acting on the particle is proportional to its angular acceleration, and the proportionality constant is the moment of inertia. It is important to note that is the rotational analogue of Newton's second law of motion, F = ma.

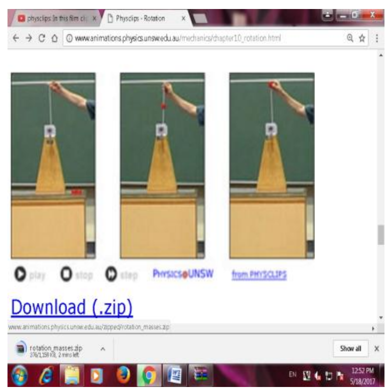
Video on relation between torque, moment of inertia and angular acceleration:

http://www.animations.physics.unsw.edu.au/mechanics/chapter10_rotation.html

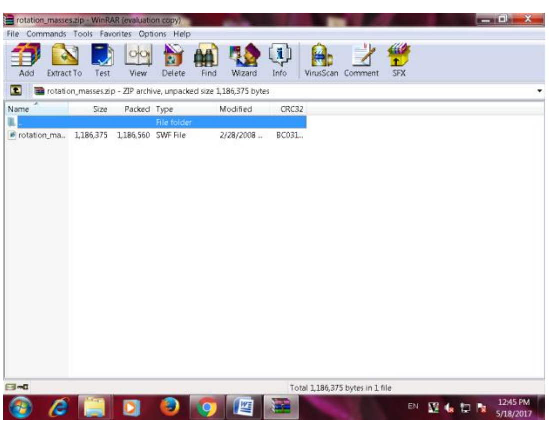
For opening this link follow the following steps:

1. **Click on the given link**
2. **Download the zip file as shown in screenshot.**

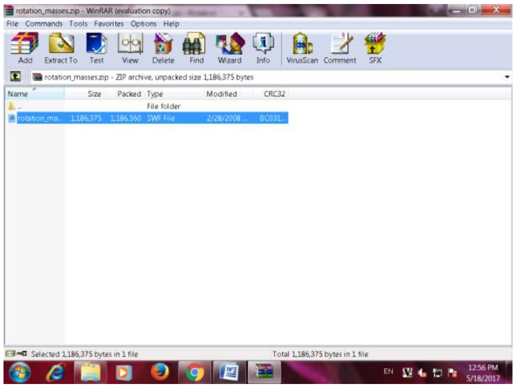




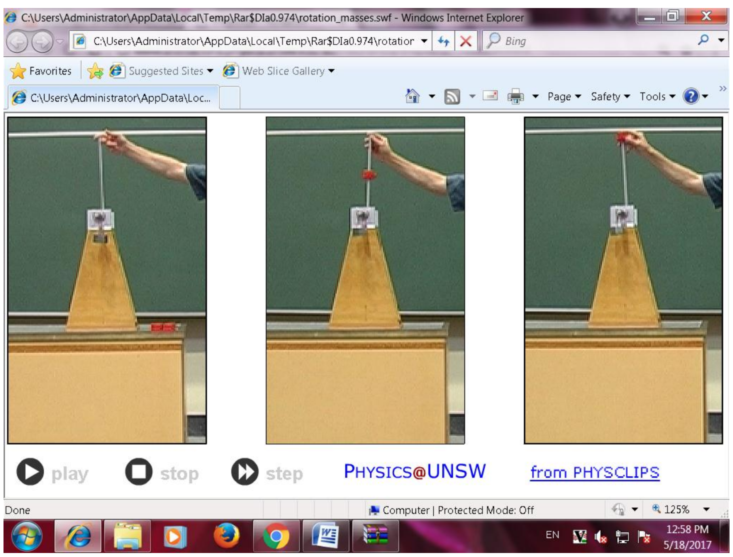
3. Extract the file by double-clicking the ZIP file to open it.



4. Click on second option as shown in above screen shot i.e., rotational masses.



5. By clicking on second option the video then open with internet explorer automatically as shown here.



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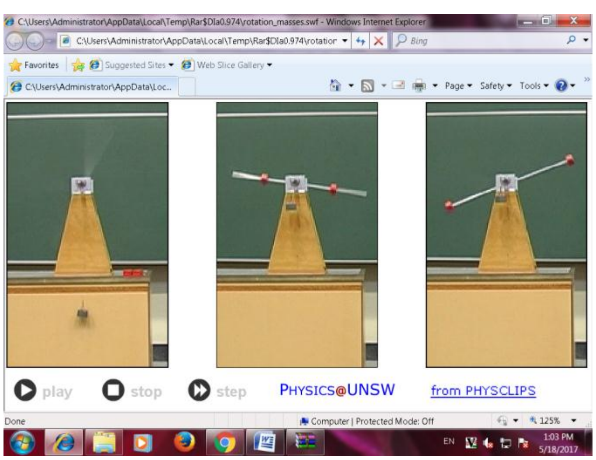
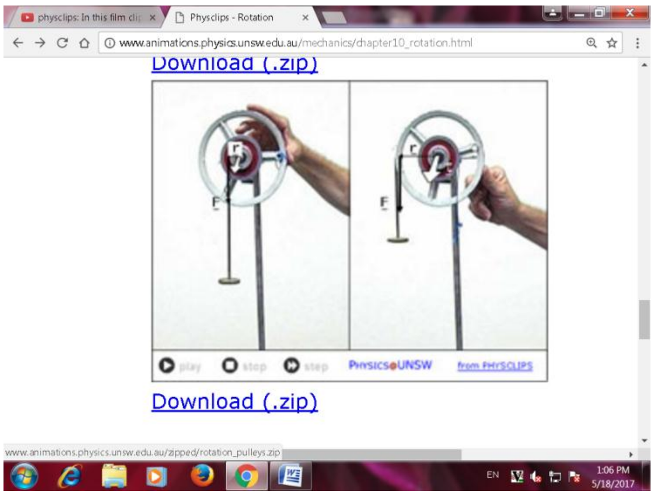
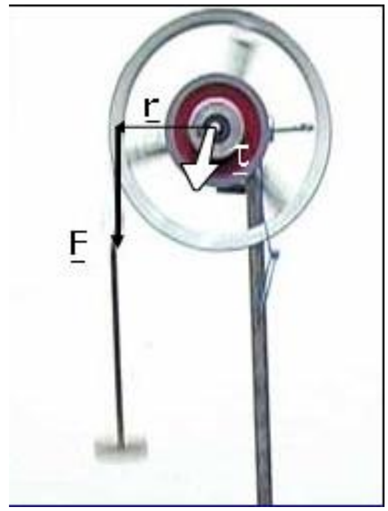
6. Click on play button to enjoy the video.



Description of videos:

In the first film, an aluminium tube rotates. In the second, masses are attached to it, increasing its moment of inertia. Larger mass, larger I, smaller α. Comparing the second and third films, we see that it is not just the mass, but also the distribution of mass that determine the moment of inertia: when the masses are present at larger radii, I is larger; α would then be smaller for a given torque τ.





In this film clip, we see different torques τ giving rise to different angular accelerations α for objects with the same moment of inertia I. Although the same mass is attached to the string, the forces are only approximately equal: the force in the example at right is a little less than that at left.

EXAMPLE:

A turn table of mass 2 kg and radius 1 m, rotates about an axis through the centre of the disc and perpendicular to the disc. The turntable is spinning at an initial constant frequency of 2 revolutions per second. The motor is turned off and the turntable slows to a stop in 6 seconds due to frictional torque. Assume that the angular acceleration (during its rotation) stays constant. What is the magnitude of the frictional torque acting on the disc?

SOLUTION:

Torque τ is related with angular acceleration α as:

$$\tau = I\,\alpha$$

Here, moment of inertia, of the disc is $I = \frac{1}{2}\,m\,r^2 = \frac{1}{2}\,2 \times (1)^2 = 1 kg\, m^2$

Also, $\alpha = \frac{\Delta\,\omega}{\Delta t} = \frac{\omega_f - \omega_i}{\Delta t} = \frac{0-(2\pi\times 2)}{6} = -\frac{2\pi}{3}\, rad\, s^{-2}$

Therefore torque due to friction is

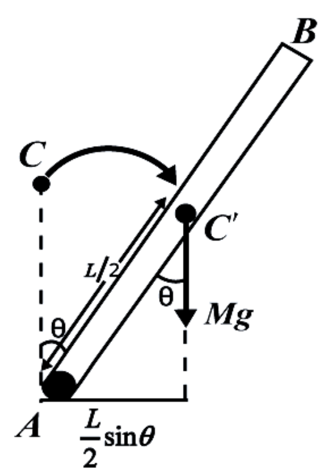
$$\tau = I\,\alpha = 1\times -\frac{2\pi}{3} = -\frac{2\pi}{3}\,\text{Nm}$$

EXAMPLE:

A slender uniform rod of mass m and length L is pivoted at one end so that it can rotate in a vertical plane. Here is negligible friction at the pivot. The free end is held almost vertically above the pivot and then releases. What is the rod's angular acceleration when it makes an angle θ with the vertical?

SOLUTION:

Let "A" point be the pivot. For calculating torque about A, let's find out how many forces are acting on the rod. There are two forces (i) weight Mg at point C and (ii) contact force at pivot.



The contact force at pivot will not contribute in torque, therefore torque due to weight will cause angular acceleration about point A.

$$\tau = I\,\alpha$$

$$\left|\vec{r} \times \vec{F}\right| = I\,\alpha$$

$$Mg\,\frac{L}{2}\,sin\theta = I\,\alpha \text{ ……………. (i)}$$

Moment of inertia about point a can be determined using parallel axis theorem as

$$I = I_c + M\left[\frac{L}{2}\right]^2$$

$$I = M\,\frac{L^2}{12} + M\left[\frac{L}{2}\right]^2 = M\,\frac{L^2}{3}\text{.........(ii)}$$

Using (i) and (ii)

$$Mg\,\frac{L}{2}\,sin\theta = M\,\frac{L^2}{3} \times \alpha$$

Therefore angular acceleration will be $\alpha = \frac{3}{2}\,g\ sin\theta$

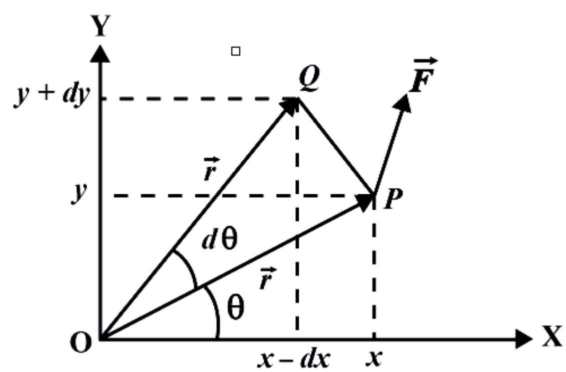
8 RECTANGULAR COORDINATE FORM OF TORQUE

Let a particle of mass m is displaced from a position P of position n vector $\vec{r_i} = x\hat{\imath} + y\,\hat{\jmath}$ to point Q of position vector $\vec{r_2} = (x + dx)\hat{\imath} + (y + dy)\hat{\jmath}$ such that $|\vec{r_i}| = |\vec{r_2}| = r$ under the influence of external force $\vec{F} = F_x\,\hat{\imath} + F_y\,\hat{\jmath}$. The particle is restricted to move in XY plane. The work done by force in displacing the particle is given as:

$$W = \vec{F}.(\vec{r_2} - \vec{r_i}) = F_x\,dx + F_y\,dy \text{ ....(1)}$$

From the figure one can write:

$x = r\,cos\theta$ and $y = r\,sin\theta$



Hence $dx = -r\,sin\theta\,d\theta = -y\,d\theta$

and $dy = r\,cos\theta\,d\theta = x\,d\theta$

Using it the equation (1) can be written as

$$W = -y\,F_x\,d\theta + F_y\,x\,d\theta = (x\,F_y - y\,F_x)d\theta$$

................(2)

Work done in rotating the given particle can be also expressed in terms of torque τ and angular displacement dθ as

$$W = \tau_z \, d\theta \quad ....(3)$$

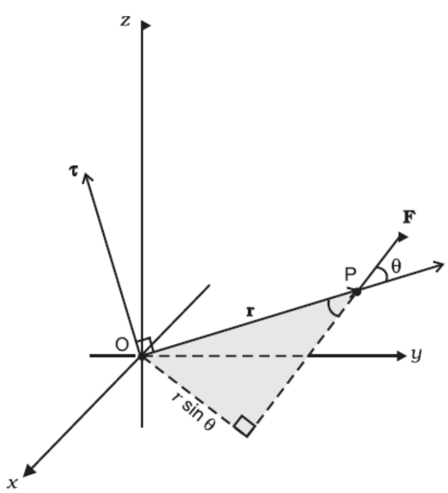
From equation (2) and (3) one can deduce torque along z axis as

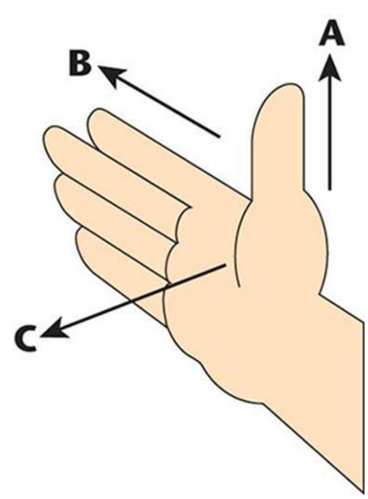
$$\tau_z = x\,F_y - y\,F_x$$

In three dimensions, this equation, coupled with the result for the cross product of two vectors, suggests that the expression for torque can be written as $\vec{\tau} = \vec{r} \times \vec{F}$

TORQUE ABOUT A POINT:

If a force acts on a single particle at a point P whose position vector is $\vec{r}$ with respect to the origin O as shown in Figure, the moment of the force acting on the particle with respect to the origin O is defined as the vector product $\vec{\tau} = \vec{r} \times \vec{F}$. The direction of torque is given with the help of Right Hand Palm rule. According to it, if $\vec{C} = \vec{A} \times \vec{B}$, the direction of $\vec{A}$ is represented by the thumb of stretched right palm and fingers represent the direction of $\vec{B}$, then direction of cross product $\vec{C}$ is represented by the normal to the direction in which the palm faces, as shown below.





When more than one forces $\vec{F_1}, \vec{F_2}, \ldots, \vec{F_n}$ are acting on a object then net torque acting on system is given as the vector sum of all the torques due to respective forces about a particular point

$$\overrightarrow{\tau_{net}} = \overrightarrow{\tau_1} + \overrightarrow{\tau_2} + \cdots \ldots + \overrightarrow{\tau_n}$$

Where $\overrightarrow{\boldsymbol{\tau_1}} = \overrightarrow{\boldsymbol{r_1}} \times \overrightarrow{\boldsymbol{F_1}}$ **is the torque due to** $\overrightarrow{\boldsymbol{F_1}}$ **and so on.**

EXAMPLE:

Find the torque of a force $(7\hat{\boldsymbol{\imath}} + 3\,\hat{\boldsymbol{\jmath}} - 5\,\widehat{\boldsymbol{k}})$ **N about the origin. The force acts on a particle whose position vector is (** $\hat{\boldsymbol{\imath}} - \hat{\boldsymbol{\jmath}} + \widehat{\boldsymbol{k}})$ **m.**

SOLUTION:

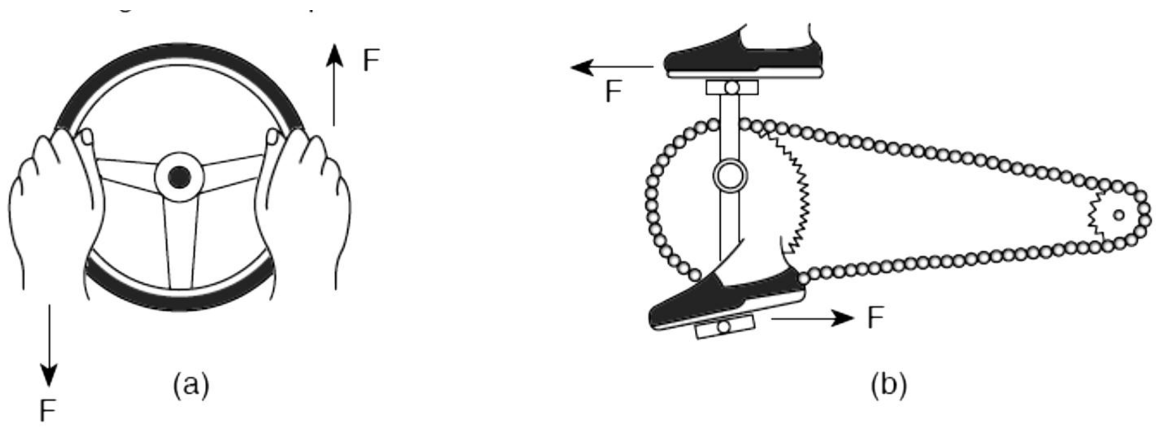
Here $\vec{r} = \ (\hat{\imath} - \hat{\jmath}\ +\ \hat{k}\ )m\ \ and\ \vec{F} = (7\hat{\imath} + 3\,\hat{\jmath} - 5\,\hat{k})$ N

Torque is given by: $\vec{\tau} = \vec{r}\ \times \vec{F}$

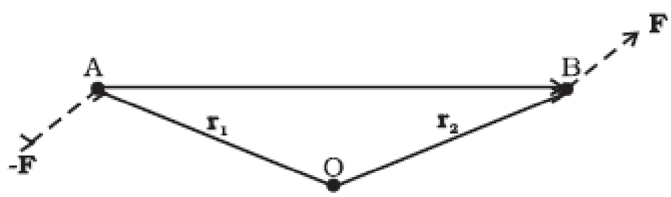
$$\vec{\tau} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & -1 & 1 \\ 7 & 3 & -5 \end{vmatrix} = 2\hat{\imath} + 12\,\hat{\jmath}\ + 10\,\hat{k}\ \ \text{Nm}$$

9. TORQUE ASSOCIATED WITH A PAIR OF FORCES, MAKING A COUPLE

A ‘Couple’ is a pair of equal and opposite forces having different lines of action. In our day-to-day life, we come across many objects which work on the principle of couple like opening and closing the cap of a bottle, turning of a water tap, using a door key, using steering wheel of a car, pedaling a bicycle etc. are all examples of situations where a pair of equal and opposite forces, i.e., a couple is used to produce the desired rotational motion. In the figure (a) below, the left hand is pulling with force F on the steering wheel while the right hand is pushing with the same force F. These two forces make a couple. Similarly in figure (b) in case of pedaling the wheels of a bicycle, the forces exerted by our feet on the two pedals again make up a couple. We are well aware of the effects of the ‘couple’ associated with these forces in the two cases.



Since the two forces are equal and opposite the net force on a system under the influence of couple is zero. The acceleration of the centre of mass will be also zero. One can conclude that a couple doesn't cause any translatory acceleration of a system. Let us consider the rotational effect of a couple on a rigid body.



Consider a couple as shown in the above Figure above acting on a rigid body. The forces and $-\vec{F}$ act at points B and A respectively. These points have position vectors $\vec{r_1}$ and $\vec{r_2}$ with respect to origin O. Let us take the moments of the forces or torque about the origin.

The moment of the couple, as a whole = sum of the moments of the two forces making the couple

$$\overrightarrow{\tau_{net}} = \vec{\tau_1} + \vec{\tau_2} = \vec{r_1} \times \vec{F} + \vec{r_2} \times -\vec{F}$$

$$= (\vec{r_1} - \vec{r_2}) \times \vec{F}$$

But

$$\vec{r_1} + \overrightarrow{AB} = \vec{r_2}$$

Hence, $\overrightarrow{AB} = \overrightarrow{r_2} - \overrightarrow{r_1}$.

The moment of the couple, is therefore,

$$\overrightarrow{\tau_{net}} = \overrightarrow{AB} \times \vec{F} .$$

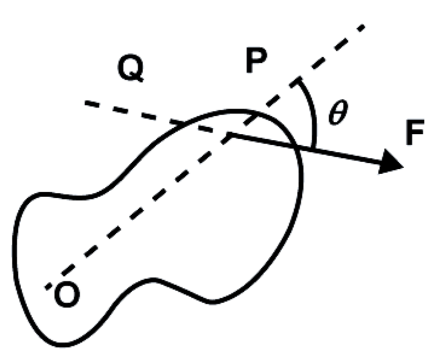
Clearly this is independent of the origin, the point about which we took the moments of the forces. The magnitude of the net torque due to a couple can also be given as:

$$\tau = \left|\overrightarrow{AB}\right| \times F$$

where $\left|\overrightarrow{AB}\right|$ is the perpendicular distance between the lines of actions of the two forces, making up the couple. Hence, the resultant torque of a couple is not zero; it is a characteristic of the couple. A couple does not produce any translation, only a rotation. It is interesting to note that moment of a couple does not depend on the point about which you take the moments. Also a couple cannot be put in equilibrium by a single force! A couple can only be put in equilibrium by a torque or by another couple of equal magnitude and opposite direction anywhere in the same plane or in a parallel plane.

CONCEPTUAL PROBLEMS:

1. **Which component of force does not contribute to its rotational effect?**
2. **Can the couple acting on a rigid body produce translatory motion?**
3. **There is a stick half of which is wooden and half is of steel. (i) it is pivoted at the wooden end and a force is applied at the steel end at right angle to its length (ii) it is pivoted at the steel end and the same force is applied at the wooden end. In which case is the angular acceleration more and why?**
4. **Graphically show how does the torque on a rigid body about O due to a given force acting at point P in the figure given below vary as:**



(i) θ is increased

(ii) The point of application of force is shifted along line of action of force PQ from P to Q.

OTHER PROBLEMS:

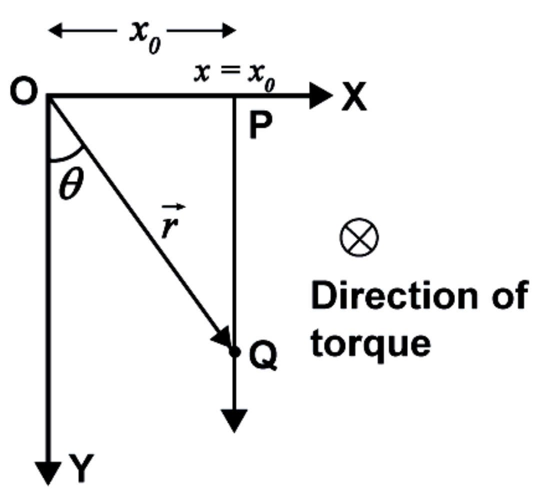
(i) A uniform disk of radius R and mass M is mounted and supported in friction less bearing A light cord is wrapped around the rim of the wheel and a steady downward pull T is exerted on the cord. Find the angular acceleration and tangential acceleration of a point on the rim.

(L = 2T/MR)

(ii) An automobile moves on a road with a speed of 54 kmh^{-1}. The radius of its wheels is 0.35 m. What is the average negative torque transmitted by its brakes to a wheel if the vehicle is brought to rest in 15s? The moment of inertia of the wheel about the axis of rotation is 3 kg m^2.

(iii) A particle is located at (4m, 6m, 0) and a force of $\vec{F} = 3\,\hat{i} + 2\,\hat{j}$ N. (a) what is the torque acting on the particle about the origin? (b) Can there be any point where torque is opposite but half in magnitude?

(iv) A particle of mass m is released from point P at x = x_O on the X axis and falls vertically along the y axis as shown in the figure. Find torque on the particle about O. (mg x_0)



a.

(v) How much torque be applied to eart in order to increase the length of day by 1 s per year? Assume that earth is a sphere of uniform density of radius 6400 km . (2.6×10^{21} Nm)

(vi) A uniform rod of length L and mass M is attached at one end to a frictionless pivot and is free to rotate about the pivot in the vertical plane, as shown in Figure. The rod is released from rest in the horizontal position. What is the initial angular acceleration of the rod?

(3g/2L)

10. SUMMARY

In this module we have learnt:

- Torque is rotational analogous of Force.
- Torque is the turning effect of the force about the given axis of rotation.
- The torque equals the moment of the force about the given axis/ point of rotation.
- Just as force equals the rate of change of linear momentum, torque equals the rate of change of angular momentum.